RADIAL DISTRIBUTION OF THE PRESSURE IN AN INDUSTRIAL REFINER (2)

Jean-Claude Roux, Georges Joris***

*Laboratoire de Génie des Procédés Papetiers, UMR 5518 au CNRS/INPG
Ecole Française de Papeterie et des Industries Graphiques
461, rue de la Papeterie, BP 65 - 38402 Saint-Martin d'Hères Cedex – FRANCE
**MATECH-EUROPE
30 rue du Gouvernement Provisoire
1000 Brussels– BELGIUM

INTRODUCTION

It concerns the refining kinetics of pulp suspensions in the low consistency refining process. The refining trials are carried out under a constant net power and a constant angular speed. This analysis is developed through a physical description of the forces encountered in an industrial refiner, forces which can contribute to the refining effects on fibres. In particular, it is interesting to anticipate the radial variability of the cutting effects on fibres, resulting from different local force intensities.
Some results, already obtained in the hydro-mechanical theory [1] will be reminded. The radial coordinate will not be first considered. Then, the description will be given, in a general sense, in order to apply for disc and conical refiners. This will lead to the formula of the global friction coefficient of the couple: cellulose materials and metal of bars.
However, as this description is not in accordance with the experimental results obtained on the cutting effects on fibres, an extension will be proposed allowing for the influence of the radial coordinate on the physical description.
Taking into account the dynamical sliding motion and the velocity of rotor bars, it will be possible to determine the net power distributed on the working zone of an industrial refiner, versus the radial coordinate. The engineering parameters will be described and their influence on the variability of the expected cutting effect on fibres will be analysed. A local extension of the reference specific edge load will be proposed and the expression of the relevant local effective pressure on fibres will be postulated.

HOW TO DETERMINE THE GLOBAL FRICTION COEFFICIENT?

Demonstration

The following analysis is general and can be extended to the case of industrial refiners running in continuous mode. If the refining trials are performed, in discontinuous mode (or batch), under constant net power $P_{net}(W)$ and constant angular velocity $\omega(rad.s^{-1})$, then the study of the influence of the net energy per unit mass of solid pulp $E_m(J.kg^{-1})$ (or net specific energy consumption) is analogous to the study of the refining kinetics through the equation:

$$E_m(t) = \frac{P_{net}.t}{m_s} \tag{1}$$

In the gap clearance of an industrial refiner, the surfaces (rotor/stator) in front of each other are fitted with bars (though it is not necessary to have bars to get refining action on fibres). Bars do not cover the complete area. Some zones represent the inter-crossing bars that give birth to the gap clearance and others (rotor bar/stator groove; rotor groove/stator bar; rotor groove/stator groove) are the complementary area. The effective fraction of the area is easily calculated by the expression:

$$\xi = \frac{a_R.a_S}{(a_R + b_R).(a_S + b_S)} \tag{2}$$

where (a,b) are respectively the width of the bars and the width of the grooves and the relevant subscripts R , S stand respectively for the rotor and the stator.

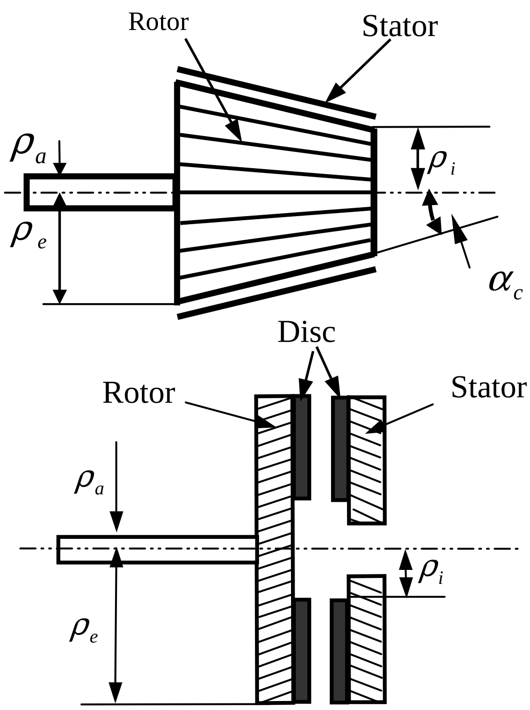

Figure 1. Configuration of a conical and a single disc refiner

Now, let us consider the space average mechanical pressure $\overline{P_m}(t)$ exerted on the compressed fibrous pads in the inter-crossing areas of the bars of a refiner (see Figure 1) and the space average hydraulic pressure $\overline{P_h}$ elsewhere. Over a small surface between the radius ρ and $\rho + d\rho$, the elementary normal force can be calculated according to the following expression:

$$dF_n(t) = \overline{P_m}(t).\xi.\frac{2\pi\rho.d\rho}{\sin(\alpha_c)} + \overline{P_h}.(1-\xi).\frac{2\pi\rho.d\rho}{\sin(\alpha_c)} \tag{3}$$

Equation (3) applies under both refining and no-refining conditions. In the case of no-refining conditions, the average hydraulic pressure is obtained anywhere (inter-crossing or the complementary areas). The equation (3) becomes:

$$dF_n^0 = \overline{P_h}.\xi.\frac{2\pi\rho.d\rho}{\sin(\alpha_c)} + \overline{P_h}.(1-\xi).\frac{2\pi\rho.d\rho}{\sin(\alpha_c)} \tag{4}$$

If the gap clearance $\overline{e}(t)$ is less than a typical value already defined [1], i.e. the critical gap clearance $\overline{e_c}$, then the elementary normal force $dF_n(t)$ is greater than the reference value dF_n^0 and the net (or effective) elementary force is defined accordingly:

$$dF_n^{eff}(t) = dF_n(t) - dF_n^0 = \left[\overline{P_m}(t) - \overline{P_h}\right].\xi.\frac{2\pi\rho.d\rho}{\sin(\alpha_c)} \tag{5}$$

The average mechanical pressure is the total pressure exerted both on the solid and the liquid phase. Either for the pressing of food materials by Körmendy and co-authors [2] or the wet pressing of paper materials by Nilsson and co-authors [3], the concept of total pressure is clearly defined. The total pressure is made up of a hydraulic pressure (on the liquid phase) and of an effective pressure, transmitted through the solid phase, in the compressed zones (inter-crossing areas of bars). The effective pressure is the expression between brackets in equation (5). From this observation, the net friction force $dF_f^{eff}(t)$, a differential quantity, can be assessed if a global friction coefficient $f(t)$ is postulated. As the morphology of the solid components of the pulp suspension depends on the refining kinetics, the global friction coefficient is obviously a function of time, then:

$$dF_f^{eff}(t) = f(t).dF_n^{eff}(t) = f(t).\left[\overline{P_m}(t) - \overline{P_h}\right].\xi.\frac{2\pi\rho.d\rho}{\sin(\alpha_c)} \tag{6}$$

In this first description of the forces exerted between rotor and stator surfaces, in an industrial refiner, the physical quantities $\overline{P_m}(t)$ and $\overline{P_h}$ are averaged on the corona located between the radii ρ_i and ρ_e (see Figure 1). Then, the integration of both equations (5) and (6) can be undertaken noting a new engineering parameter for sake of simplification:

$$k = \frac{\rho_i}{\rho_e} \tag{7}$$

After integration over the complete corona and replacing this ratio by the new parameter k , it leads to:

$$\begin{cases} F_n^{eff}(t) = \left[\overline{P_m}(t) - \overline{P_h}\right].\xi.\dfrac{\rho_e^2\left(1-k^2\right)}{\sin(\alpha_c)} \\ F_f^{eff}(t) = f(t).\left[\overline{P_m}(t) - \overline{P_h}\right].\xi.\dfrac{\rho_e^2\left(1-k^2\right)}{\sin(\alpha_c)} \end{cases} \tag{8}$$

These effective elementary forces in (5) and (6), or the resulting power consumption by friction, are responsible for the refining effects to occur. The differential friction power can be determined through the following equations:

$$dP_f^{eff}(t) = dF_f^{eff}(t).\omega.\rho = f(t).\left[\overline{P_m}(t) - \overline{P_h}\right].\xi.2\pi\omega\frac{\rho^2.d\rho}{\sin(\alpha_c)} \tag{9}$$

If one postulates that this friction power, integrated over the complete corona, is the net power consumed for the refining effects, then an average radius $<\rho>$ can be calculated according to:

$$P_f^{eff} = P_{net} = f(t).F_n^{eff}(t).\omega.<\rho> \tag{10}$$

then:

$$<\rho> = \frac{2\rho_e\left(1-k^3\right)}{3\left(1-k^2\right)} \tag{11}$$

From equations (8) and (10), it is possible to extract the global friction coefficient $f(t)$. We have:

$$\boxed{f(t) = \frac{3.P_{net}.\sin(\alpha_c)}{\xi.2\pi\omega.\rho_e^3.\left(1-k^3\right).\left[\overline{P_m}(t) - \overline{P_h}\right]}} \tag{12}$$

This global friction coefficient leads to a non-determination when the net power equals zero since the average total pressure is equal to the average hydraulic pressure and no refining effects are visible on fibres.
In the paper industry, the cutting effect on fibres is often related to an intensity index called the specific edge load. In a previous work, we have generalized this concept through the reference specific edge load since this index correctly accounts for the influence of engineering variables on the cutting effect on fibres:

$$C_s^0 = \frac{3.P_{net}.(a_R + b_R)(a_S + b_S).\sin(\alpha_c)}{2\pi\omega.\rho_e^3(1-k^3)} \tag{13}$$

If we combine equations (12) and (13), it leads to a fundamental result that gives the physical meaning of the reference specific edge load:

$$\boxed{\overline{P_m}(t) - \overline{P_h} = \frac{C_s^0}{f(t).a_R.a_S}} \tag{14}$$

Commentaries

This equation (14) reveals that only average quantities can give birth to the reference specific edge load, a quantity of an equally average nature. However, the refining kinetics can be developed through a hydrodynamic description, considering a complex fluid of an equivalent apparent viscosity, as done in reference [4]. It can be demonstrated that the mechanical pressure is a function of the space coordinate through the refiner, as it is in any lubricated bearing. In this hydrodynamic description, the use of averaged variables does not allow to be close to the reality.

By extension, in a conical or a disc refiner, the mechanical pressure should be a function of the radial coordinate in the direction of pulp motion, from the internal to the external radius. This suggests that the reference specific edge load should be a local quantity or, at least, a function of the radial coordinate: $C_s^0(\rho)$. If we are able to evaluate this local quantity, then the mechanical pressure $P_m(\rho,t)$ will be defined by an extension of equation (14) accordingly:

$$P_m(\rho,t) - \overline{P_h} = \frac{C_s^0(\rho)}{f(t).a_R.a_S} \tag{15}$$

A question remains to be solved: how to determine the local reference specific edge load? Let us suppose that an elementary refiner has a working area delimited by the radii ρ and $\rho + d\rho$. In that case, the reference specific edge load should be defined as follows:

$$C_s^0(\rho) = \frac{dP_f^{eff}(\rho,t)}{dL_c^0(\rho)} \tag{16}$$

where the quantity, in the denominator, represents the elementary cutting speed given by the expression:

$$dL_c^0(\rho) = \frac{2\pi\rho}{a_R + b_R}.\frac{2\pi\rho}{a_S + b_S}.\frac{\omega}{2\pi}.\frac{d\rho}{\sin(\alpha_c)} \tag{17}$$

If the effective power is dissipated by friction, as calculated by equation (9), then it can be shown that the reference specific edge load does not appear to be a function of the radial coordinates ρ. From this statement, the calculation of the power dissipated must be undertaken differently. In the following paragraph, a particular attention will be paid to the sliding velocity of moving bars in the rotating motion of the rotor in front of the stator.

KINEMATICS OF THE ROTOR BARS AND SLIDING VELOCITY

Analysis of the kinematics (case of a single disc refiner)

The radial coordinate is now considered and the case of a single disc refiner has been selected to illustrate the kinematics. On Figure 2, we can see 2 positions of a given rotor bar at time t and time $t+dt$. Our purpose is to determine the *local sliding velocity* of this given moving bar since it reveals a displacement, materialized by the distance AB, during the time dt.

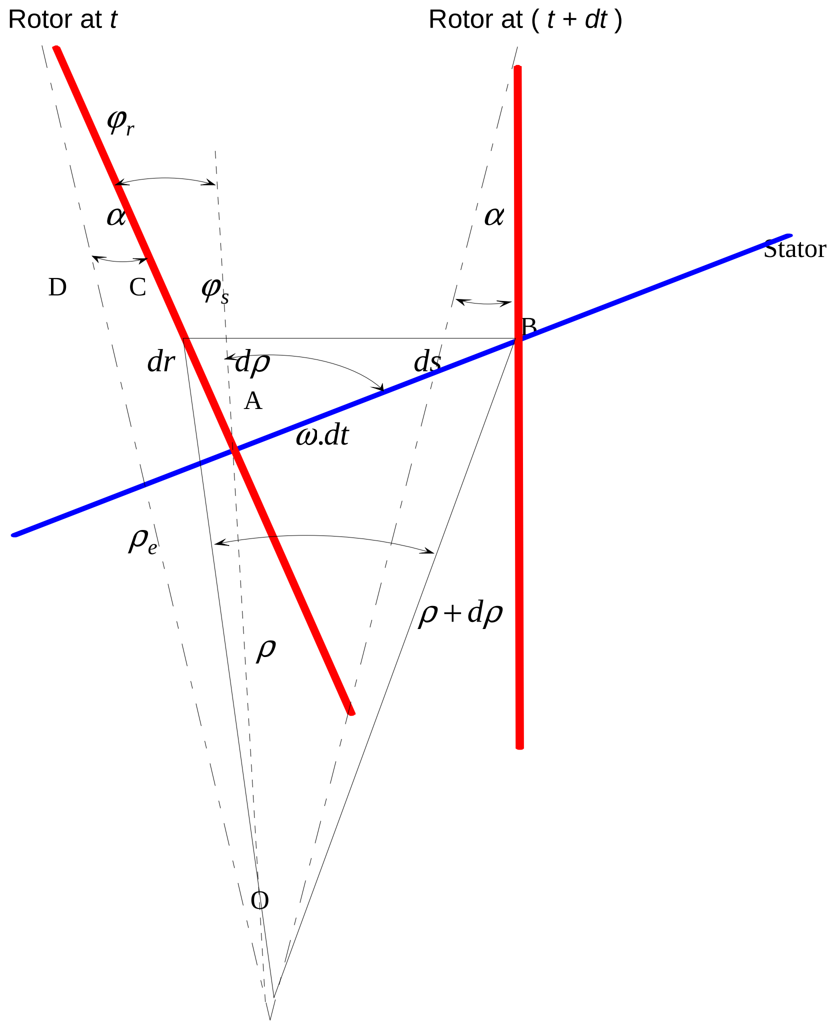

Figure 2. Motion of a given rotor bar in front of a given stator bar in the case of a disc refiner

Since the distance OA represents the coordinate ρ and the distance OB the coordinate $\rho + d\rho$, it is possible to estimate $d\rho$ by the distance AC. What is the physical meaning of the distance AD? It is the sliding displacement of the point A on the rotor bar during the elementary time dt. These sliding displacements AB and AD are differential quantities that can be calculated in considering the two local angles φ_s, φ_r respectively, of the given stator bar (rotor bar respectively) versus the radial direction $\overrightarrow{OC}$. In the rectangular triangles ACB and ACD, the following differential equations can be easily obtained:

$$\begin{cases} d\rho = AB.\cos(\varphi_s) = ds.\cos(\varphi_s) \\ d\rho = AD.\cos(\varphi_s) = dr.\cos(\varphi_r) \end{cases} \tag{18}$$

Since the measure of the angle DOB is given by $\omega.dt$, then the distance DB can be determined by the followings, considering the first order in the developments:

$$\left(\rho + d\rho\right).\omega.dt = ds.\sin\left(\varphi_s\right) + dr.\sin\left(\varphi_r\right) \cong \rho.\omega.dt \tag{19}$$

By combining the previous set of equations (18) and (19), and noting γ the local crossing angle of the given bars (see Figure 2), we obtain:

$$\rho.\omega.dt = ds.\frac{\sin\left(\varphi_r + \varphi_s\right)}{\cos\left(\varphi_r\right)} = ds.\frac{\sin\left(\gamma\right)}{\cos\left(\varphi_r\right)} \tag{20}$$

If the fibres (to be impacted) cover the stator bars as it was observed by some authors, it is sensible to pay a greater attention to the sliding velocity of a given rotor bar on a given stator bar. The sliding velocity is calculated as follows:

$$V\left(\rho,\gamma,\alpha\right) = \frac{ds}{dt} = \frac{\omega.\rho.\cos\left(\varphi_r\right)}{\sin\left(\gamma\right)} = \frac{\omega.\sqrt{\rho^2 - \rho^2.\sin^2\left(\varphi_r\right)}}{\sin\left(\gamma\right)} = \frac{\omega.\sqrt{\rho^2 - \rho_e^2.\sin^2\left(\alpha\right)}}{\sin\left(\gamma\right)} \tag{21}$$

The last expression introduces the local angle α of the relevant rotor bar. However, everything being equal, the sliding velocity is not the same depending on the first rotor bar or the last one on the same sector angle θ is considered. To calculate the effective power dissipated by friction, all rotor bars versus all stator bars should be considered. We chose another way, postulating that the geometrical angular parameters could be given by their average values. In that case, this “average” velocity $V\left(\rho,\overline{\gamma},\overline{\alpha}\right)$ is an increasing function of the local radius ρ.

The problem of assessing the effective power dissipated by friction during the bar motion can be approached easily.

Distribution of the cumulated effective power

Under the previous assumptions, the net power can be written by the following expression with an unknown average angle $\lambda(t)$ between the tangential force $dF_f^{eff}\left(\rho,t\right)$ and the average sliding velocity $V\left(\rho,\overline{\gamma},\overline{\alpha}\right)$:

$$dP_f^{eff}\left(\rho,t\right) = dF_f^{eff}\left(\rho,t\right).\frac{\omega.\sqrt{\rho^2 - \rho_e^2.\sin^2\left(\overline{\alpha}\right)}}{\sin\left(\overline{\gamma}\right)}.\cos\left(\lambda(t)\right) \tag{22}$$

By replacing the differential friction force by equation (6) and putting $\alpha_c = \pi/2$ for the case of a single disc refiner, another expression can be obtained:

$$dP_f^{eff}\left(\rho,t\right) = \left\{f(t).\left[\overline{P_m}(t) - \overline{P_h}\right]\xi.2\pi\omega\right\}.\frac{\rho.\sqrt{\rho^2 - \rho_e^2.\sin^2\left(\overline{\alpha}\right)}}{\sin\left(\overline{\gamma}\right)}.\cos\left(\lambda(t)\right).d\rho \tag{23}$$

Then, the expression between brackets can be replaced with the help of equation (12), it lasts:

$$dP_f^{eff}\left(\rho,t\right) = \frac{3.P_{net}}{\rho_e^3\left(1-k^3\right)}.\frac{\cos\left(\lambda(t)\right)}{\sin\left(\overline{\gamma}\right)}.\rho.\sqrt{\rho^2 - \rho_e^2.\sin^2\left(\overline{\alpha}\right)}.d\rho \tag{24}$$

In this equation, an angle remains unknown, this can be done through boundary conditions since the power is distributed from 0 on the internal radius ρ_i until P_{net} on the external radius ρ_e. The equation (24) allows calculating the distribution of the *cumulated effective power* dissipated by friction from the internal radius ρ_i until a local radius ρ:

$$\mathsf{P}_{eff}(\rho)=\int_{\rho_i}^{\rho} dP_f^{eff}=\frac{P_{net}}{\rho_e^3\left(1-k^3\right)}.\frac{\cos(\lambda(t))}{\sin(\overline{\gamma})}.\left\{\left[\rho^2-\rho_e^2.\sin^2(\overline{\alpha})\right]^{3/2}-\left[\rho_i^2-\rho_e^2.\sin^2(\overline{\alpha})\right]^{3/2}\right\} \quad (25)$$

After simplifications and introducing the engineering ratio *k* defined by equation (7), the unknown angle can be obtained from the boundary limits, this leads to the general result for the cumulated effective power from the internal until the local radius:

$$\mathsf{P}_{eff}(\rho)=P_{net}.\frac{\left[\left(\frac{\rho}{\rho_e}\right)^2-\sin^2(\overline{\alpha})\right]^{3/2}-\left[k^2-\sin^2(\overline{\alpha})\right]^{3/2}}{\cos^3(\overline{\alpha})-\left[k^2-\sin^2(\overline{\alpha})\right]^{3/2}} \quad (26)$$

and also leads to the definition of the λ angle and of the constant $A(k,\overline{\alpha})$ as follows:

$$\cos(\lambda)=\frac{\left(1-k^3\right)}{\cos^3(\overline{\alpha})-\left[k^2-\sin^2(\overline{\alpha})\right]^{3/2}}.\sin(\overline{\gamma})=A.\sin(\overline{\gamma}) \quad (27)$$

This angle is not time dependant and the time *t* must be vanished in the previous equation (25). In order to give a physical meaning to this result, let consider radial bars for the rotor disc, then the lambda angle is easily determined, it is the complementary of the average crossing angle $\left(\lambda=(\pi/2)-\overline{\gamma}\right)$.

Distribution of the reference specific edge load

In the general case (not only radial bars for the rotor), the *local reference specific edge load* can be obtained for a single disc refiner $\left(\sin(\alpha_c)=1\right)$ by the following expressions, deduced from equations (16), (17) and (25) until (27):

$$\boxed{C_s^0(\rho)=\frac{dP_f^{eff}(\rho,t)}{dL_c^0(\rho)}=\frac{d\mathsf{P}_{eff}(\rho)}{dL_c^0(\rho)}=C_s^0.\frac{\left(1-k^3\right)}{\cos^3(\overline{\alpha})-\left[k^2-\sin^2(\overline{\alpha})\right]^{3/2}}.\frac{\sqrt{(\rho/\rho_e)^2-\sin^2(\overline{\alpha})}}{(\rho/\rho_e)}} \quad (28)$$

It can be demonstrated and observed on Figure 3 that the local reference specific edge load $C_s^0(\rho)$ is an increasing function of the radial coordinate ρ. The variability of the foreseen cutting effect on fibres can be assessed, for example, by the ratio *R* of the maximum value (obtained on the external radius) to the minimum value (obtained on the internal radius):

$$R=\frac{C_s^0(\rho_e)}{C_s^0(\rho_i)}=\frac{\left(\cos\overline{\alpha}\right).k}{\sqrt{k^2-\sin^2\overline{\alpha}}} \quad (29)$$

From a theoretical point of view, a refiner for which *k = 1* gives no variability to the local reference specific edge load or no variability on the cutting effect on fibres. This refiner with an internal radius equals to the external one is a *cylindrical refiner*.
As this technology is not well spread in Paper Industry, on both theoretical and practical points of view, it is more useful to evaluate the importance of the variability for conical and disc refiners.

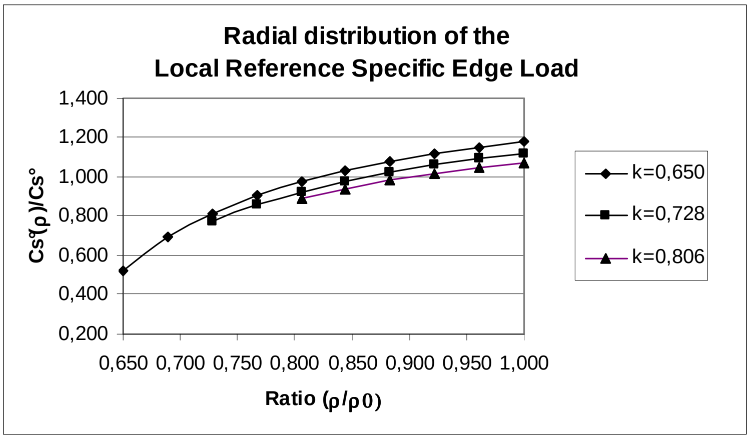

Figure 3 – Influence of the k ratio on the radial distribution
of the Local Reference Specific Edge Load for $\overline{\alpha} = 37{,}5°$

If a *conical refiner* is used with an average bar angle $\overline{\alpha}$ equal to *0*, then the ratio *R* is equal to *1* whatever is the value of k, since a simplification occurs. No variability is foreseen for the cutting effect on fibres in that case.
For a *disc refiner* with a typical value of k, the ratio of the local reference specific edge loads is increasing with the average bar angle $\overline{\alpha}$. Equation (28) reveals the existence of a limiting value for this angle where the ratio *R* can reach high numerical values:

$$\overline{\alpha}_{\lim} = Arc\sin(k) \tag{30}$$

For example, the ratio *R* is equal to 1.57 with $\alpha = 15°$; $\theta = 30°$; $k = 0.60$; $\overline{\alpha} = 30°$; $\overline{\alpha}_{\lim} = 37°$
Hence, the local reference specific edge load on the external radius is 57% higher than that of the internal radius. The knowledge of the local reference specific edge load is sufficient to determine the pressure. The radial pressure $P_m(\rho,t)$ is given by the extension of equation (14) for the case of *a "theoretical" refiner* with a running area comprised between the radii ρ and $(\rho + d\rho)$:

$$\boxed{P_m(\rho,t) - \overline{P_h} = \frac{C_s^0(\rho)}{f(t).a_R.a_S}} \tag{31}$$

The same conclusions apply accordingly. It is demonstrated that the relative pressure is an increasing function of the radial coordinate. The difference between the maximum and minimum pressure is given by:

$$P_m(\rho_e) - P_m(\rho_i) = \frac{C_s^0(\rho_i)}{f.a_R.a_S}.\left[\frac{(\cos\overline{\alpha}).k}{\sqrt{k^2 - \sin^2\overline{\alpha}}} - 1\right] \tag{32}$$

This difference is increasing with the average angle $\overline{\alpha}$, but is decreasing with *k*. The most homogeneous results on the cutting effect on fibres are obtained with a refiner where both internal and external radii are identical, i.e. *a cylindrical refiner*. In the case of *conical refiners* where all bar angles are equal to 0, the total pressure applied on the pulp pads, located in the inter-crossing areas of bars, is uniform versus the radial coordinate.
For the case of *disc refiners*, the total relative pressure, or the local pressure transmitted through the solid phase, is an increasing function of the radial coordinate exemplified by equations (28) and (31). It remains a difference in the pressure applied which can be assessed between the internal radius and the external one.
These theoretical findings are experimentally verified at the industrial scale.

REFERENCES

[1] J.C. Roux, J. Chu, G. Joris, G. Caucal. Théorie hydromécanique du fonctionnement d'un raffineur à basse concentration. *Revue A.T.I.P.* **53**(4-5) : pp 106-116, 1999.

[2] J.C. Roux. Stock Preparation Part 1 – Pulp treatment processes. In **Fibres and their Preparation**, Trans. 12th Fund. Res. Symp., (ed. C.F. Baker), pp 19-80, 2001.

[3] I. Körmendy, Contribution to the three dimensional pressing theory and its one-dimensional application. *Acta Alimentaria* **3**(1): pp 93-110, 1974.

[4] P. Nilsson and K.O. Larsson. Paper web performance in a press nip. *Pulp and Paper Mag. of Canada* **69**(24): pp 66-73, 1968.

[5] J.C. Roux, G. Joris, G. Caucal. Angular parameters beyond specific edge load. In proc. *6th Pira International Refining Conference*, Paper 9, Toronto, Canada, Pira Editions, Leatherhead, 2001.